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ONE DIRECT DIAGRAM OF A METHOD OF CHARACTERISTICS FOR THE CALCULATION OF A THREE DIMENSIONAL GAS FLOW

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ONE DIRECT DIAGRAM OF A METHOD OF CHARACTERISTICS FOR THE CALCULATION OF A THREE DIMENSIONAL GAS FLOW

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ABSTRACT. In the method proposed characteristic surfaces are constructed and in place of trigonometric representation of parameters by one variable, a finite differential approximation along a certain vector lying in the characteristic surface is used. Derived information is ordered according to one of the variables distributed along meridional planes, using a cylindrical system of coordinates. In comparison with data of other methods using the results of calculation of supersonic flow on a spherically truncated cone at an angle by initial data on the characteristic surface shows that the new method more accurately reflects the prevalent physical particulars. It can also be adopted to extrapolate calculation from the initial data or to obtain the initial data for other methods on non-characteristic surfaces, and provides a precise physical flow picture.

Of the quantitative methods in use which are based on the use of characteristics, one can sort out two groups. The first group is made up of essentially characteristic methods, their characteristic surfaces are constructed in the solution process [1, 2]. The so-called tetrahedral or prismatic diagrams can be allotted to this group. In the tetrahedral diagram, three known points are used to obtain a fourth. This diagram is developed in [1] and practically applied in [2]. Unfortunately, in [2] the shock wave is calculated incorrectly (see [3]). The use of three known points in an elementary cell to obtain a fourth allows a wide latitude in the selection of the diagram. The idea of a prismatic diagram can be found for example, in [4, 5]. /1413*

Here we propose a direct method of characteristics, which is a direct generalization of a two-dimensional method [6]. Derived information is ordered according to one of the variables (distributed in meridional planes), which distinguishes the given method from that proposed in [1, 2]. This diagram

*Numbers in the margin indicate pagination in the foreign text.

differs from the one used in [7] by being direct, i.e., in the calculation process characteristic surfaces are constructed and in place of trigonometric representation of parameters by one variable, a finite differential approxi- /1414
mation along a certain vector lying in the characteristic surface is used. Generally speaking, the diagram is implicit in that direction.

For unknown functions we will examine pressure p, enthalpy i and two corners of velocity vector V in a cylindrical system of coordinates x, r, ϕ ($V = V[(1 + \eta^2)(1 + \zeta^2)]^{-1/2}\{1, \eta, \zeta\sqrt{(1 + \eta^2)}\}$). The unknown functions are relative to input parameters (p is relative to $\rho_\infty V_\infty^2$, i--k V_∞^2), linear dimensions to a specific dimension of a body (for example, to the radius of a sphere); x is read off from the forward point of the body; angle ϕ is read off so that the input flow velocity vector V_∞ lies in the plane $\phi = 0$. We have $V_\infty = \{\cos\alpha, -\sin\alpha\cos\phi, \sin\alpha\sin\phi\}$ where α is the angle of incidence. The initial system of equations establishing the ideal flow of gas in these variables is given in [8]. We will convert this system to work out a numerical diagram.

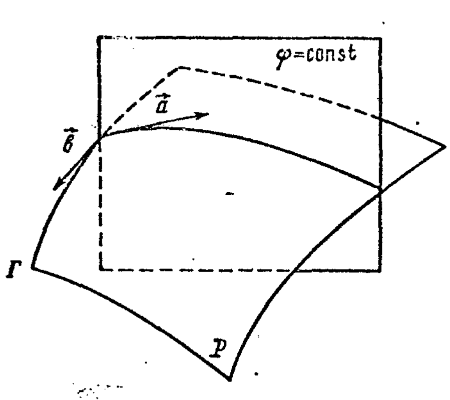

Figure 1.

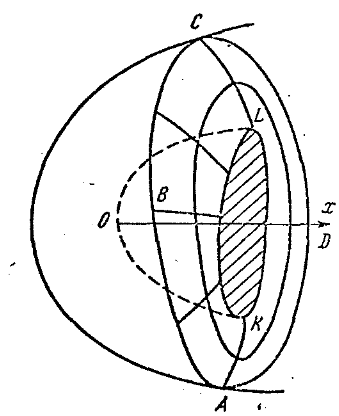

Figure 2.

Suppose we are given continuous curve Γ of the three-dimensional type with tangential unit vector b (Figure 1). Through curve Γ we direct characteristic plane P. We direct unit vector a, tangent to surface P and lying in plane ϕ = const, i.e., $\alpha_\phi = 0$. We resolve a and b into tangent vectors $\tau(t)$ and $\sigma(t)$ of surface P, where $\tau(t)$ is directed along the bicharacteristics, $\sigma(t)$ is perpendicular to $\tau(t)$ and to the characteristic normal $n(t)$ (see [9], and $t \in [0, 2\pi]$. Evidently the following correlations take place.

$$\mathbf{a} = \boldsymbol{\tau} \cos \mu + \boldsymbol{\sigma} \sin \mu, \qquad \mathbf{b} = \boldsymbol{\tau} \sin \nu + \boldsymbol{\sigma} \cos \nu, \qquad \mathbf{a}\mathbf{n}(t) = 0, \qquad \mathbf{b}\mathbf{n}(t) = 0. \tag{1}$$

From the last correlation we find parameter T. Note that this equation has two roots in the interval $t \in [0, 2]$ for wave planes and one for the flow plane. From the third equation we find the vector $a = \{\sin \lambda, \sin \lambda; 0\}$. Then corners ν and μ are determined by correlations $\cos \mu = a\tau$, $\sin \mu = a\sigma$, $\cos \nu = b\sigma$, $\sin \nu = b\tau$. Then the equations common to the wave planes can be written in the form

$$C_1 \frac{\partial \eta}{\partial s} + C_2 \frac{\partial \zeta}{\partial s} + C_3 \frac{\partial P}{\partial s} = D_1 \frac{\partial \eta}{\partial l} + D_2 \frac{\partial \zeta}{\partial l} + D_3 \frac{\partial P}{\partial l} + F \cos(\nu + \mu), \tag{2}$$

where
$$C_1 = \frac{\sin t \cos \nu - \sin \varepsilon \cos t \sin \nu}{(1+\eta^2)\sqrt{(1+\zeta^2)}}, \qquad D_1 = \frac{\sin t \sin \mu - \sin \varepsilon \cos t \cos \mu}{(1+\eta^2)\sqrt{(1+\zeta^2)}},$$

$$C_2 = \frac{\cos t \sin \mu + \sin \varepsilon \sin t \cos \mu}{1+\zeta^2}, \qquad D_2 = -\frac{\cos t \cos \mu - \sin \varepsilon \sin t \cos \mu}{1+\zeta^2},$$

$$C_3 = \frac{\Lambda}{\operatorname{tg} \varepsilon} \cos \nu, \qquad D_3 = \frac{\Lambda}{\operatorname{tg} \varepsilon} \sin \mu, \qquad F = \frac{(\zeta \cos t - \operatorname{tg} \varepsilon)\eta \sqrt{(1+\zeta^2)} + \zeta^2 \sin t}{r(1+\zeta^2)\sqrt{[(1+\eta^2)(1+\operatorname{tg}^2 \varepsilon)]}}.$$

and those common to the flow plane

$$\begin{gathered} C_1^0 \frac{\partial \eta}{\partial s} + C_2^0 \frac{\partial \zeta}{\partial s} + C_3^0 \frac{\partial P}{\partial s} = D_1^0 \frac{\partial \eta}{\partial l} + D_2^0 \frac{\partial \zeta}{\partial l} + D_3^0 \frac{\partial P}{\partial l} + F^0 \cos(\nu + \mu), \\ \frac{\partial I}{\partial s} - \frac{1}{Z} \frac{\partial P}{\partial s} = \frac{\sin \mu}{\cos \nu} \left(\frac{\partial I}{\partial l} - \frac{1}{Z} \frac{\partial P}{\partial l} \right), \end{gathered} \tag{3}$$

where
$$C_1^0 = \frac{\cos t \cos \nu}{(1+\eta^2)\sqrt{(1+\zeta^2)}}, \qquad D_1^0 = \frac{\cos t \sin \mu}{(1+\eta^2)\sqrt{(1+\zeta^2)}},$$

$$C_2^0 = \frac{\sin t \cos \nu}{1+\zeta^2}, \qquad D_2^0 = \frac{\sin t \sin \mu}{1+\zeta^2},$$

$$C_3^0 = -\Lambda \sin \nu, \qquad D_3^0 = -\Lambda \cos \mu,$$

$$F^0 = \frac{\zeta[\zeta \cos t - \eta \sin t \sqrt{(1+\zeta^2)}]}{r(1+\zeta^2)\sqrt{(1+\eta^2)}}.$$

Here s is the coordinate (length of the arc) along vector a, along vector b, /1415
$\Lambda = p/\rho V^2$, $Z = \rho i/p$, $P = \ln p$, $I = \ln i$, $\tan \varepsilon = (M^2 - 1)^{-1/2}$, M is the local Mach number. These equations are also the initial ones for setting up the numerical diagram.

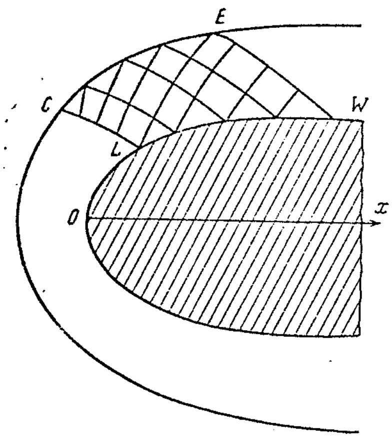

Figure 3

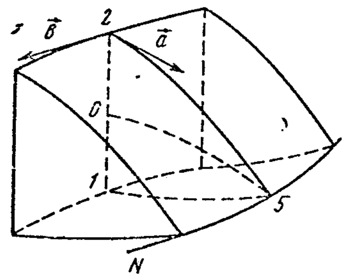

Figure 4

Now we have the problem of numerical solution of the equations for the gas dynamics in the supersonic region limited by the previously unknown shock wave and the body according to the initial data on a certain characteristic surface ABCDLK (Figure 2).

We will construct a numerical diagram which is a direct inference of the two dimensional characteristic [6]. The products along vector b are replaced with finite differential ratios. A second (or first) family calculating lattice CL is constructed, and step by step the solution is found in the region bounded by unknown shock wave CE and body surface LW. In each plane ϕ = const we will have a picture as shown in Figure 3. The surfaces fixed by CL and LE are characteristic.

Now examine the elementary core of differential lattice (Figure 4). Equations (2), taken for two values of parameter t, are described along lines 1-5 and 2-5, while equations (3) are along 0-5. These curves are the essence of the intersection of the corresponding characteristic surfaces with planes ϕ = const, and have equations which are written in differential form

$$\frac{r_5 - r_i}{x_5 - x_i} = \operatorname{tg} \lambda_i, \qquad i = 0, 1, 2, \tag{4}$$

where tan λ is defined by the third equation of (1).

Now we write equations (2) and (3) in the differential form:

$$C_1^i \eta_5 + C_2^i \zeta_5 + C_3^i P_5 = \Sigma_i, \qquad i = 0,\ 1,\ 2,$$
$$I_5 = I_0 + \frac{P_5 - P_0}{Z} + \frac{\sin\mu}{\cos\nu}\frac{\Delta s}{\Delta l}\left[(I_+ - I_-) - \frac{P_+ - P_-}{Z}\right], \tag{5}$$

where

$$\Sigma_i = C_1^i \eta_i + C_2^i \zeta_i + C_3^i P_i + \frac{\Delta s}{\Delta l}[D_1^i(\eta_+ - \eta_-) + D_2^i(\zeta_+ - \zeta_-) + D_3^i(P_+ - P_-)] + F^i \cos(\mu + \nu).$$

Subscript "5" relates to the parameter in lattice point 5; the subscripts "-" and "+" indicate that the functions belong to the preceding or subsequent planes respectively with respect to the subject plane $\phi = \phi_0$. Qualitatively, the values $f\pm$ take average values of the corresponding parameters in points i and 5 in planes $\phi_0 = \Delta\phi$. Coefficients C_j^i and D_j^i in equation (5) and tan λ_i in equation (4) are computed according to the average values of the parameter in points i.

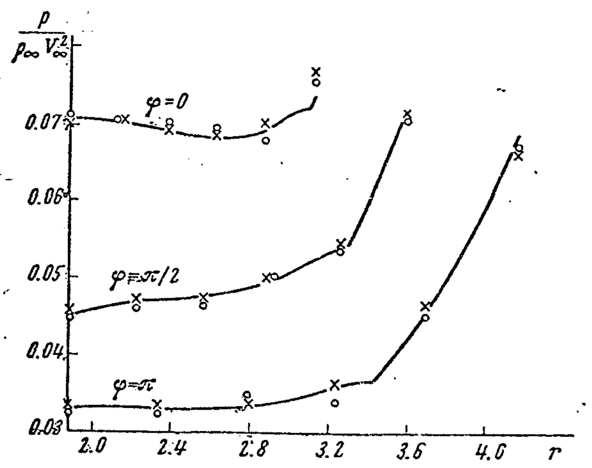

Figure 5.

Let the initial data be known for characteristic surfaces of the second family. Then we begin the calculation of each layer from the shock-wave point. If the initial data are known for the characteristics of the first family, then calculation begins with the point for the body. The shock wave is sought in the form $f = R(x, \phi)$.

Calculation of points in the shock wave. Take the intersection normal to the shock wave $n = (1 + q^2 + \omega^2)^{-1/2}$ $\{-q, 1, \omega\}$ and two tangent unit vectors $a = (1 + q^2)^{-1/2}\{1, q, 0\}$, $b = \{b_x, b_r, b_\phi\}$, where $q = R_x$, $\omega = R_\phi/R$, b is the known vector.

Corresponding to [3], using (5) where i = 1 and the condition nb = 0, we obtain the equation for determining q_5:

$$C_1{}^1\eta(q_5) + C_2{}^1\zeta(q_5) + C_3{}^1P(q_5) - \Sigma_1 = 0. \tag{6}$$

Carrying the first family characteristic out to intersection with the shock wave, we find the coordinates of point 5. Then, solving equation (6), we find q_5, and from correlation nb = 0, we determine ω_5, we further calculate in the first approximations of parameters η_5, ζ_5, P_5, and I_5 the points of the shock wave for all ϕ by known formulas (see [8]). Then for a zero approximation we take the parameters at the preceding layer. Then we repeat the calculations using neutralized parameters.

Calculating points within the flow. From points 1 and 2 (Figure 4) we carry out the characteristics (in the first approximation for parameters in points 1, 2) to intersection at point 5. Solving system (4), we determine the parameters η_5, ζ_5, P_5, I_5. Then the initial values for the flow line is taken according to the neutralized parameters in points 1 and 2, and then, carrying the flow line backwards from point 5, we find point 0 and the parameters in it of the linear interpolation between points 1 and 2. After calculating all points 5 in line N (Figure 4) for all ϕ in the first approximation, we repeat the calculating process, this time calculating the coefficients by the center values of the parameters in points i and 5 (i = 0, 1, and 2). /1417

Calculating the points on the body. Let the body be given by equation r = $= r_T(x, \phi)$. Then the condition of nonincursion on the body can be written in the form

$$\eta_5 = r_{Tx}' + \zeta_5 \sqrt{(1+\eta_5^2)} \frac{r_{T\varphi}'}{r_T}.$$

The method of calculation is analoguous to the preceding one, but in place of correlation along line 1-5 one must use the nonincursion condition of the body. /1418

Calculation was made according to the algorithm described above for the supersonic region between the shock wave and the body from a certain initial characteristic surface of the second family.

To verify the method we calculated the flow around a cone with a half angle of ϕ = 10° and with spherical truncated at an angle of incidence of α = = 5°, while M_∞ = 6. Calculations were made for 25 points in the characteristic and 11 points on the angular coordinate. Initial data were taken from the case of flow around a sphere [10], and then the system of coordinates was rotated to angle α. Figure 5 illustrates the pressure profile transverse to the shock layer in cross-section x = 5.8264 for three planes $\phi = 0, \pi/2, \pi$. Here is clearly seen the break in the product along the characteristic surface going from the break line in the curved contour of the body. Comparison was made with the lattice-characteristic method, wherein the data from [9] are indicated by "°" and from [11] by "x".

Let us look at the results of calculation of flow around a round cylinder truncated along an ellipsoid rotation with a semiaxial ratio of $\delta = b/a = 1.5$ for $\alpha = 5°$ and $10°$ and $M_\infty = 10$. Initial data for this case are taken from calculations of the subsonic and transsonic region given in [12]. Linear dimensions relate to the greater semiaxis b.

To extend the calculation into the supersonic region we took 17 points on the characteristic and 11 points on the angular coordinate. Figure 6 shows the flow picture in the plane of symmetry ($\alpha = 10°$). Figure 7 gives the distribution of pressure along the surface of the cylinder in three planes $\phi = 0, \pi/2, \pi$ ($\alpha = 5°$).

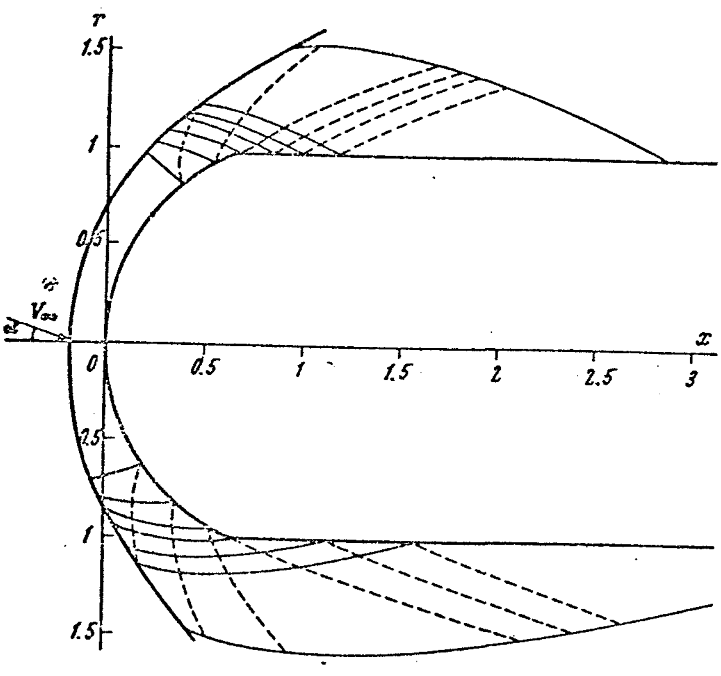

Figure 6.

By this method we have proposed and in practice realized a new direct diagram of characteristics for calculation of three-dimensional gas flow. In constructing characteristic surfaces points are ordered along one of the variables (distributed on the meridional planes). Comparison of the results of calculating supersonic flow on a spherically truncated cone at an angle of incidence by initial data on the characteristic surface with the data of other methods showed that the method

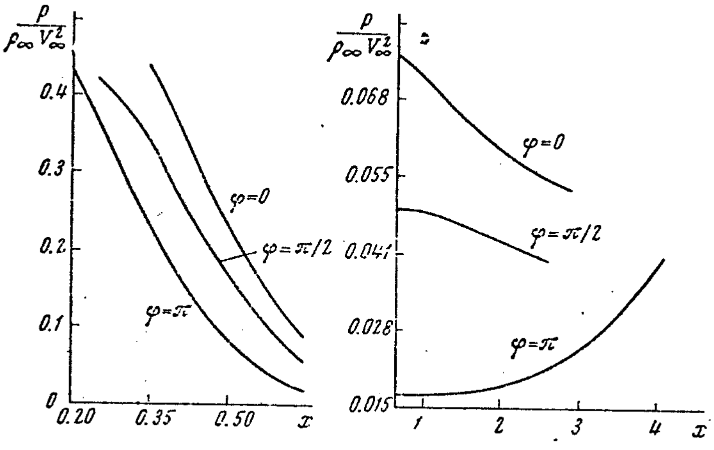

Figure 7.

suggested is a more accurate reflection of the prevalent physical particulars.

It is useful to adopt the method given if it is necessary to extrapolate calculation from the initial data on the characteristic surface or to obtain the initial data for other methods on non-characteristic surfaces, and also if one wishes to get a precise physical flow picture.

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